



## Studies on resilience of water networks

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### ABSTRACT

Water system integration is one of the most efficient technologies for saving fresh water and reducing wastewater. However, the compact connection of water units often leads to insufficient resilience of a water system. In order to counteract the fluctuation of a real production process, a water network must be flexible. Two new concepts, maximum tolerance amount of a water unit (MTaWU) and tolerance amount of a water unit (TaWU) are introduced firstly in this paper. Based on these two concepts, the tolerance amount of a water network (TaWN) is proposed to quantify the resilience of a water network. A case study illustrates that these parameters are countable and straightforward.

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### 1. Introduction

Techniques involving water network synthesis are generally categorized as the water pinch design method [1–4] and the mathematical programming approach [5–7]. The water network design is intended to achieve the minimum water targets, however, water network design process assumes that process data are fixed and well-defined, whereas the actual operating conditions such as water flowrate and corresponding mass loads may fluctuate over time. These fluctuations in processing conditions can lead to process disruptions or product quality problem. So we must consider network resilience or flexibility during water network designing. The resilience of a water network can be comprehended as that when some parameters, such as mass load, fluctuate within a certain operating range, the whole water network can also satisfy the operation.

Grossmann and Morari [8] firstly presented the concept of resilience in process industries; and they defined resilience as the ability of a plant to tolerate and to recover from dynamic or transient disturbances. Such as the HEN can response the dynamic disturbance of temperature and flowrate smoothly. There are three kinds of index to evaluate the resilience of a network at present. Saboo et al. [9] proposed the resilience index (RI) to characterize the largest total uncertainty which a HEN can tolerate while remaining feasible. Swamy and Grossmann [10] introduced a flexibility index (FI), which defines the maximum parameter range that can be achieved for feasible steady-state operation. Ierapetritou [11] presented a new method, which is called as feasible convex

hull ratio (FCHR) to evaluate the resilience, which is based on the interconnected feasible region and its size.

The resilience studies of networks mainly concentrate on the HEN. Colberg and Morari [12] incorporated the resilience concept within a synthesis approach for flexible HEN, proposing a “flexibility index target”. Cerda et al. [13] proposed a different synthesis methodology for obtaining flexible HEN, introducing the concept of “transient” and “permanent” streams in order to describe variations in inlet temperatures and flow rates. Floudas and Grossmann [14,15] proposed a sequential HEN synthesis method that combines the multiperiod mixed-integer linear programming (MILP) transshipment model with the active set strategy to guarantee the desired HEN flexibility. Li [16] presented a method using multiple linear equations to design the resilience of HEN.

There is little about the resilience or flexibility analysis and design of water networks available. Wang [17] proposed a method of water network design under uncertain parameters. It involves two methods: fixed resilience design and optimal flexibility design. The author also presented the principle of selecting prime modulation design parameters according to sensitivity analysis theory. Du [18] analyzed the resilience of water network and the design of water network involving resilience by calculating the resilience index and the FCHR. Tan et al. [19] use the Monte Carlo simulation to assess the sensitivity of water networks to noisy mass loads. The method can select the most robust network configuration from three alternation designs. Lama et al. [20] introduced a preliminary controllability and resiliency analysis for a whitewater network. The proposed approach is available to analyze the real dynamics of the process.

However, resilience or flexibility studies on resilience of water network analysis and design are still at theoretical phrase presently. The main reasons that prevent the practical application of resilience

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### Nomenclature

$C_{i,out}$	outlet concentration of unit $i$ (ppm)
$C_{i,in}^{max}$	maximum inlet concentration of unit $i$ (ppm)
$C_{i,out}^{max}$	maximum outlet concentration of unit $i$ (ppm)
$F_i$	total water flowrate through unit $i$ (t/h)
$F_i^W$	freshwater flowrate into unit $i$ (t/h)
$F_{i,j}$	water flowrate from unit $i$ to $j$ (t/h)
$M_i$	TaWU of unit $i$ (kg/h)
$M_i^{max}$	MTaWU of unit $i$ (kg/h)
$\bar{M}$	TaWN of process $i$ (kg/h)
MTaWU	maximum tolerance amount of a water unit
OBNU	outflow branch number of unit
RU	rank of unit
$R_L$	the RU of the least TaWU
Ta	tolerance amount
TaWU	tolerance amount of a water unit
TaWN	tolerance amount of a water network

### Greek letters

$\eta_i$	OBNU coefficient of unit $i$
$\mu_i$	RU coefficient of unit $i$

### Subscripts

$i$	number of water unit in a water network
$n$	total number of water units in a water network

research are as follows. First, the presented approaches or models are too complicated, which are hard to apply in complex system. Second, nearly all of related issues are highly nonlinear, there are not effective resolving techniques.

Hence, this paper presents another more simple approach to quantify the resilience of a water network (WN). This method is straightforward and easily calculated.

## 2. The tolerance amount of a water unit

For an existing water network, if the inlet and outlet concentrations are less than their limiting concentrations, water using processes will meet the operation requirements. Therefore, to estimate the flexibility of an existing water network, we only need to consider the relative size between limiting inlet and outlet concentrations and the actual concentrations.

If the outlet concentration of a water unit is less than the limiting outlet concentration, then the operation is quite normal when the mass load changes within a certain range. The surplus ability for removing contaminants of such a unit is designated as tolerance amount (Ta). A unit has resilience if the unit has non-zero Ta, and likewise, the Ta of a water network can also be apprehended as the resilience of the water network. Therefore, the resilience of a network can be qualified through calculating the Ta and estimating whether the distribution of Ta is reasonable.

Water networks are made up of water units, so MTaWU is defined firstly.

If the outlet contaminant concentration of a unit is less than the limiting outlet concentration, the product of the total water flowrate of the unit and the difference between the limiting outlet concentration and the outlet concentration, is defined as MtaWU. The formula of MTaWU are specified as:

$$M_i^{max} = F_i \times (C_{i,out}^{max} - C_{i,out}) \quad i = 1, 2, \dots, n \quad (1)$$

where  $M_i^{max}$  is denotation of MtaWU;  $F_i$  is the water flowrate of unit  $i$ ;  $C_{i,out}^{max}$  is the limiting outlet concentration of unit  $i$ ;  $C_{i,out}$  is the actual outlet concentration of unit  $i$ .

In an existing water network, if the contaminant mass load of a certain water unit increases while those of the other relevant units before this unit remain unchanged, MTaWU is the maximum value, which can be achieved by this unit. So the MTaWU can be comprehended as the cumulation of all of relevant units before; and also shows the transfer characteristic of Ta.

The TaWU, which is defined as the MTaWU subtracting from the cumulation of Ta, is cumulated by all of relevant units before the unit.

There are three basic network structure, namely, series, concurrent, and branch, which can form any types of water network structures, as shown in Fig. 1. Different structures have different formulas of TaWU, as given in Eqs. (2)–(4).

### Series mode

$$\begin{aligned} M_1 &= M_1^{max} \\ M_i &= M_i^{max} - M_{i-1}^{max} \quad i = 2, 3, \dots, n \end{aligned} \quad (2)$$

### Concurrent mode

$$M_n = M_n^{max} - \sum_{i=1}^{n-1} M_i^{max} \quad (3)$$

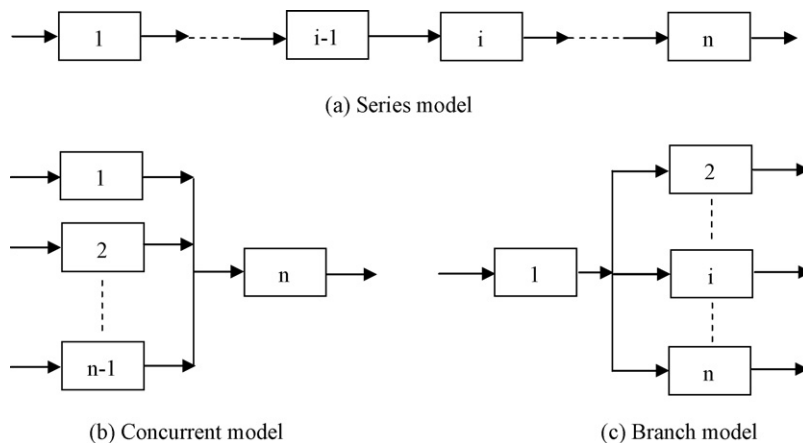


Fig. 1. Three basic structures of water using network.

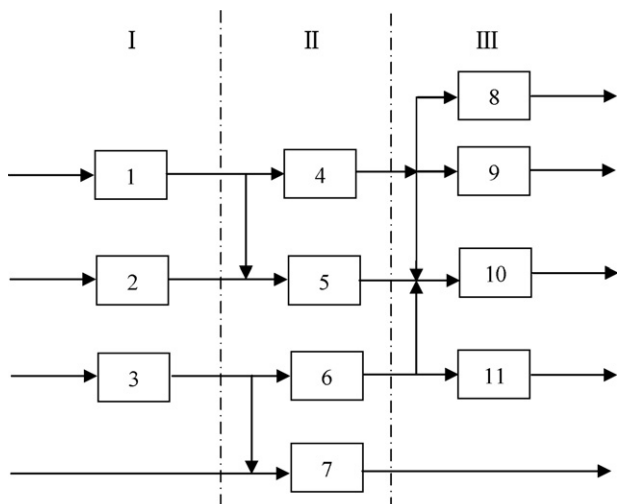


Fig. 2. A water system to show RU.

Branch mode

$$M_i = M_i^{\max} - \frac{F_{1 \rightarrow i}}{F_1} \times M_1^{\max} \quad i = 2, 3, \dots, n \quad (4)$$

TaWU can be described as the maximum surplus ability to remove contaminant of the unit itself for a given operating conditions. When the mass load in the unit increases, if the outlet concentration of the unit reaches the limiting outlet concentration, the maximum variation of the mass load of the unit can be apprehended as TaWU.

### 3. The tolerance amount of a water network

A water network consists of a number of water units. The outlet concentrations of a unit, which is decided by the outlet concentrations of the upstream units and water using condition of itself, must be less than the limiting outlet concentrations.

If the outlet concentrations of each unit reach the limiting outlet concentration, which means that there is no Ta in this water network at such operating conditions, the mass load in the water network is the limiting mass load. The network has Ta when the outlet concentrations of one or more units are less than the limiting concentrations. The value of Ta can be interpreted as the difference between the limiting mass load of the network and the actual mass load.

For two units that have the same Ta, if their position in the network is different, the influence to the network Ta is also different. So we cannot just simply add the Ta of each unit to calculate the Ta of the whole network. Therefore, determining Ta of a network should consider the Ta of each unit and the position of each unit in the network. Thus, two concepts are defined, which are rank of unit (RU) and outflow branch number of unit (OBNU).

The rules to define the RU are as follows: A unit that only uses fresh water is defined as first-degree unit, a unit that accepts the used water from the first-degree unit is defined as second-degree unit, and the rest can be deduced by analogy. If a certain unit accepts water from different degree units, its degree is based on the largest degree of all those units whose water enter this unit, namely, the degree of this unit is one more than of the largest degree.

A water system is shown in Fig. 2. The first-degree units are units 1–3. The second-degree units are units 4–7. The third-degree units are units 8–11. The RU can represent the relative position of each unit in a whole network and the water quality that accept from other units.

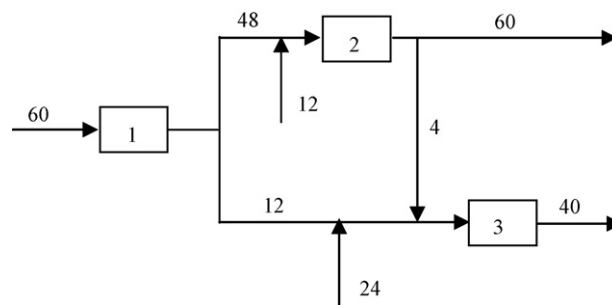


Fig. 3. The sketch of the water using system.

The OBNU is defined as the total outflow stream branch number (the effluent streams are excluded) plus one. The OBNU is the unit number that the Ta of the unit can affect. Because the Ta of a unit can influence itself, the OBNU should add one, and at the same time, the effluent stream to the wastewater main has no effect to the other unit which should not be included. Following this definition, in Fig. 2, the OBNU of units 1, 3 and 6 is 3, the OBNU of units 2 and 5 is 2, the OBNU of unit 4 is 4, the OBNU of units 7–11 is 1. The OBNU reflects the range that the Ta of a unit can affect, namely, the number of units that can influence.

Based on these two concepts, a formula of the Ta of a water network (TaWN) is proposed to determine the resilience of the water network. In this formula, the RU and the OBNU are considered as two weighting factors, to include the contribution of each TaWU. Usually, the unit which has less Ta is always the resilience bottleneck of water networks, here, the unit which has least Ta is considered.

$$\bar{M} = \frac{\sum \mu_i \eta_i M_i}{\sum \mu_i \eta_i + R_L^2} \quad i = 1, 2, \dots, n \quad (5)$$

The two coefficients are determined as following rules.

- (1) If the RU is 1, 2, ..., N, then  $\mu_1 = N$ ,  $\mu_2 = N - 1$ , ...,  $\mu_n = 1$ .
- (2) If the OBNU of unit  $i$  is  $m$ , then  $\eta_i = m$ .

The TaWN is the weighted average value of all the water units in a water network. Supposed that the mass load of each water unit increases by the same quantities at the same time, the maximum possible variation of the mass load of each unit is the TaWN.

## 4. Case study

### 4.1. Example 1

A water network of a chemical plant is shown in Fig. 3, which has three units. The data for this system are shown in Table 1. Using the data in Table 1, the actual outlet concentration of each unit can be calculated. Then based on Eq. (1), the MTaWU can be determined, which is  $M_i^{\max}$  shown in Table 2. The detailed calculation is

Table 1  
The water using data of the chemical plant

Units	$F_{ij}$ (t/h)	$C_{i,out}$ (ppm)	$C_{i,out}^{\max}$ (ppm)	$F_i^W$ (t/h)
1	0	105	120	60
2	$F_{1,2} = 48$	113	130	12
3	$F_{1,3} = 12, F_{2,3} = 4$	114	140	24

**Table 2**  
The Ta and coefficients of water units

Units	$M_i^{\max}$ (kg/h)	$M_i$ (kg/h)	$\mu_i$	$\eta_i$
1	0.9	0.9	3	3
2	1.02	0.3	2	2
3	1.04	0.8	1	1

as follows:

$$M_1^{\max} = F_1 \times (C_{1,\text{out}}^{\max} - C_{1,\text{out}}) = 60 \times (120 - 105) \times 10^{-3} = 0.9$$

$$M_2^{\max} = F_2 \times (C_{2,\text{out}}^{\max} - C_{2,\text{out}}) = (48 + 12) \times (130 - 113) \times 10^{-3} = 1.02$$

$$M_3^{\max} = F_3 \times (C_{3,\text{out}}^{\max} - C_{3,\text{out}}) = (12 + 4 + 24) \times (140 - 114) \times 10^{-3} = 1.04$$

According to the network structure and the corresponding equations (Eqs. (2)–(4)), the TaWU of each unit can also be specified, which is  $M_i$  shown in Table 2. The detailed calculation is as follows:

$$M_1 = M_1^{\max} = 0.9 \text{ kg/h}$$

$$M_2 = M_2^{\max} - \frac{F_{1 \rightarrow 2}}{F_1} \times M_1^{\max} = 1.02 - \frac{48}{60} \times 0.9 = 1.02 - 0.70 = 0.3 \text{ kg/h}$$

$$M_3 = M_3^{\max} - \frac{F_{1 \rightarrow 3}}{F_1} \times M_1^{\max} - \frac{F_{2 \rightarrow 3}}{F_2} \times M_2^{\max} = 1.04 - \frac{12}{60} \times 0.9 - \frac{4}{60} \times 1.02 = 1.04 - 0.18 - 0.06 = 0.8 \text{ kg/h}$$

The rank weighting coefficient and the OBNU weighting coefficient are also included in Table 2.

The TaWU of unit 1 can effect units 1–3, so OBNU of unit 1 is 3. Consider the Ta data in Table 2. For unit 1, there is not Ta which is transferred from other upstream units, so the MTaWU and the TaWU are same as 0.9 kg/h. For unit 2, the TaWU is 0.3 kg/h, and the MTaWU is 1.02 kg/h, because there is Ta transferred from unit 1. Similarly, as the third-degree unit, the unit 3 has the Ta from the unit 1 which is first-degree unit and the unit 2 which is the second-degree unit, so the TaWU is 0.8 kg/h, and at the same time, the MTaWU is 1.04 kg/h.

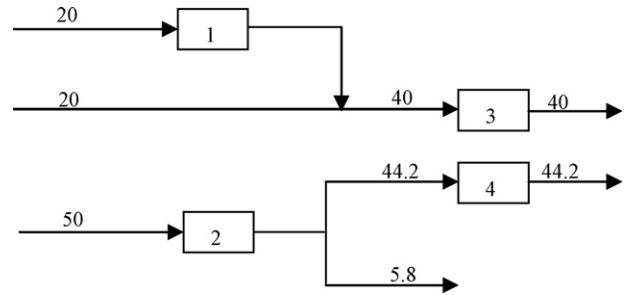
Using the data in Table 2, the TaWN can be specified by using Eq. (5) as follows.

$$\bar{M} = \frac{\sum \mu_i \eta_i M_i}{\sum \mu_i \eta_i + R_L^2} = \frac{3 \times 3 \times 0.9 + 2 \times 2 \times 0.3 + 1 \times 1 \times 0.8}{3 \times 3 + 2 \times 2 + 1 \times 1 + 2^2} = 0.56 \text{ kg/h}$$

The TaWN can be comprehended as the mean value of the Ta of all the units in a water network, so the average Ta of this network is 0.56 kg/h. Then it is seen that what will happen if the mass load of each unit increases 0.56 kg/h. When the mass load of unit 1 increases 0.56 kg/h, the MTaWU decreases to 0.34 kg/h. According to Eq. (4), the Ta of unit 1 transfers 0.27 kg/h to unit 2 and 0.07 kg/h to unit 3, respectively. So the MTaWU of unit 2 achieves to 0.57 kg/h. Similarly, if the mass load of the unit 2 increases 0.56 kg/h, the Ta of unit 2 is approximate to 0. For unit 3, by adding the Ta transferred from units 1 and 2, the MTaWU becomes 0.87 kg/h, if the

**Table 3**  
Limiting process water data (based on Wang and Smith [1])

Process number	$C_{i,\text{in}}^{\max}$ (ppm)	$C_{i,\text{out}}^{\max}$ (ppm)	Water flowrate (t/h)	Design mass load (kg/h)	Normal mass load (kg/h)
1	0	100	20	2	1.5
2	50	100	100	5	4.4
3	50	800	40	30	29
4	400	800	10	4	3.6



**Fig. 4.** Water network for minimum of water sources (Wang and Smith [1]).

mass load of unit 3 increases 0.56 kg/h, and its Ta become 0.31 kg/h. The working condition of the whole water network is very well.

#### 4.2. Example 2

In order to illustrate the Eq. (5) more clearly, another more established case study is used. The example is introduced by Wang and Smith [1]. The limiting process water data are shown in Table 3. Based on these data, Wang and Smith have used the water pinch diagram achieved two networks, one for the maximum driving force, and the other for the minimum number of water sources. Fig. 4 is the water network for minimum number of water sources. Unlike the original case study, the mass loads are assumed to be variable and the normal mass loads are shown in Table 3. These normal mass loads can be interpreted as mean mass load of each process. The actual inlet and outlet concentration can be determined by using the data in Table 3, as shown in Table 4. Based on the normal mass loads and the specified water network, just like example 1, the MTaWU and the TaWU can be specified, as shown in Table 5. The rank weighting coefficient and the OBNU weighting coefficient are also included in Table 5. Finally, the TaWN of Fig. 4

**Table 4**  
The actual process water data based on normal mass load

Process number	$C_{in}$ (ppm)	$C_{out}$ (ppm)	Water flowrate (t/h)	Normal mass load (kg/h)
1	0	75	20	1.5
2	0	88	50	4.4
3	37.5	762.5	40	29
4	88	709	5.8	3.6

**Table 5**  
The Ta and coefficients of water units

Units	$M_i^{\max}$ (kg/h)	$M_i$ (kg/h)	$\mu_i$	$\eta_i$
1	0.5	0.5	2	2
2	0.6	0.6	2	2
3	1.5	1.0	1	1
4	0.47	0.4	1	1

can be specified as follows.

$$\bar{M} = \frac{\sum \mu_i \eta_i M_i}{\sum \mu_i \eta_i + R_L^2} = \frac{2 \times 2 \times 0.5 + 2 \times 2 \times 0.6 + 1 \times 1 \times 1 + 1 \times 1 \times 0.4}{2 \times 2 + 2 \times 2 + 1 \times 1 + 1 \times 1 + 2^2} = 0.41 \text{ kg/h}$$

Using the same analysis method, the average Ta of water network is 0.41 kg/h, so we can see what will happen if the mass load of each unit increases 0.41 kg/h. The MTaWU of the four units is 0.09, 0.19, 0.68 and 0, respectively.

## 5. Conclusions

Through defining Ta, the problem of evaluating the resilience of a water network is transformed into that of calculating the TaWN. This methodology is simple and straightforward. Example in the paper shows the feasibility and practicability of the method when evaluating the resilience of water networks.

Although the approach is used in single contaminant systems, it can be applied to multiple contaminants systems, if a key contaminant is chosen. Meanwhile, this method can only be used to evaluating the existing network, rather than designing a new one.

The concepts proposed in this paper can be extended to other systems, such as hydrogen network. Such work will be explored in the future.

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